Name: ____

Braden River High School

<u>Algebra 2</u>

Review Packet

For students entering Algebra 2H in August 2016

Directions:

1. In order to take full advantage of the video instruction offered by the web links provided in each section, <u>this packet is best completed while opened on your computer</u>.

2. Complete all exercises and bring this packet to your initial Algebra 2H class in August 2016. You may use a calculator (except when noted otherwise); but, show all work used to arrive at your answer. Expect to be quizzed on this material on a day to be announced.

3. If you need help on any of the Algebra1 topics in this packet, please refer to the following websites:

- http://coolmath.com/algebra/Algebra1/index.html
- http://www.algebra.com/
- http://www.brightstorm.com/math
- http://www.khanacademy.org

Table of Contents

1. Order of Operations

Order of Operations – Evaluating an Expression

http://www.brightstorm.com/math/algebra/pre-algebra/order-of-operations
http://www.khanacademy.org/video/order-of-operations?playlist=Developmental%20Math

Use PEMDAS (Please Excuse My Dear Aunt Sally):
Step 1. P – Parentheses: start with operations inside grouping symbols (parentheses)
Step 2. E – Exponents: evaluate powers
Step 3. MD – Multiply/Divide: do multiplications and divisions from left to right
Step 4. AS – Add/Subtract: do additions and subtractions from left to right

Ex1)

$$15 \bullet 2 \div 6$$
$$(15 \bullet 2) \div 6$$
$$30 \div 6 = 5$$

Ex2)

$$3 \bullet (4^2 + 8) \div 4$$
$$3 \bullet (16 + 8) \div 4$$
$$3 \bullet 24 \div 4$$
$$72 \div 4 = 18$$

Ex3)

$$\frac{7 \bullet 4}{8 + 7^2 - 1} = \frac{7 \bullet 4}{8 + 49 - 1}$$
$$= \frac{28}{57 - 1}$$
$$= \frac{28}{56}$$
$$= \frac{1}{2}$$

1. $8 \div \frac{1}{2} \bullet 3 + (6 - 4)$	2. $\frac{(3+2)(-8)}{(-3)^2 + 1}$	3. $3\left[2(4+1)^2\right] - 10^2$
4. $\left[(9-7)^2 + 5\right] + 26$	5. $\frac{8 \bullet 2 + 5}{12 + 2^2 - 9}$	6. $8 + 4^3 \div 8 - 3$

2 Simplifying Radicals

Use Properties of Radicals to Simplify Expressions
http://www.brightstorm.com/math/algebra/radical-expressions-and-equations/simplifying-radical-expressions
http://www.khanacademy.org/video/simplifying-radicals?playlist=Pre-algebra

Product Property: The square root of a product equals the product of the square roots of the factors

$$\sqrt{ab} = \sqrt{a} \bullet \sqrt{b}$$

Ex1) $\sqrt{400} = \sqrt{4 \bullet 100} = \sqrt{4} \bullet \sqrt{100} = 2 \bullet 10 = 20$

Ex2) $\sqrt{75} = \sqrt{25 \bullet 3} = \sqrt{25} \bullet \sqrt{3} = 5\sqrt{3}$

Quotient Property: The square root of a quotient equals the quotient of the square roots of the numerator and denominator

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

Ex3) $\sqrt{\frac{9}{25}} = \frac{\sqrt{9}}{\sqrt{25}} = \frac{3}{5}$

Ex4) $\sqrt{\frac{27}{16}} = \frac{\sqrt{27}}{\sqrt{16}} = \frac{\sqrt{9 \bullet 3}}{4} = \frac{\sqrt{9} \bullet \sqrt{3}}{4} = \frac{3\sqrt{3}}{4}$

Simplest Form: An expression with radicals is in simplest form if the following are true:

- no perfect square factors (other than 1) are in the radicand (under the radical): $\sqrt{8} \rightarrow \sqrt{4 \bullet 2} \rightarrow 2\sqrt{2}$
- no fractions are in the radicand (under the radical): $\sqrt{\frac{5}{16}} \rightarrow \frac{\sqrt{5}}{\sqrt{16}} \rightarrow \frac{\sqrt{5}}{4}$
- no radicals appear in the denominator of the fraction: $\frac{1}{\sqrt{5}} \rightarrow \frac{1}{\sqrt{5}} \bullet \frac{\sqrt{5}}{\sqrt{5}} \rightarrow \frac{\sqrt{5}}{\sqrt{25}} = \frac{\sqrt{5}}{5}$

[This is called "rationalizing the denominator": http://www.khanacademy.org/video/how-to-rationalize-a-denominator?playlist=ck12.org%20Algebra%201%20Examples]

Simplify – leave answers in simplest radical from - no decimal answers.

1. $\sqrt{120}$ **2.** $4\sqrt{90}$ **3.** $10\sqrt{250}$ **4.** $\sqrt{32}$

5. $\sqrt{80}$ **6.** $\sqrt{125}$ **7.** $\sqrt{\frac{9}{81}}$ **8.** $\sqrt{\frac{1}{169}}$

9. $\sqrt{\frac{20}{16}}$ **10.** $\sqrt{\frac{9}{5}}$ **11.** $\sqrt{\frac{10}{15}}$ **12.** $\sqrt{\frac{60}{7}}$

3 Linear Equations

A. **Graphing.** The graph of a **linear equation** is a line. Here is a quick review of how to graph common forms of a linear equation.

Form 1: Horizontal Line $y = c\,(\text{c} = \text{real number})$

The graph of $y = c$ is a horizontal line that passes through the point $(0,\ c)$. The equation $y = 5$ is simply graphed by moving on the y-axis to the point $(0,\ 5)$ and then drawing a horizontal line (see figure 1). Likewise, the equation $y = 0$ is graphed by going to the point $(0,\ 0)$ and drawing a horizontal line – in this case, the graph is the *x*-axis (see figure 2).

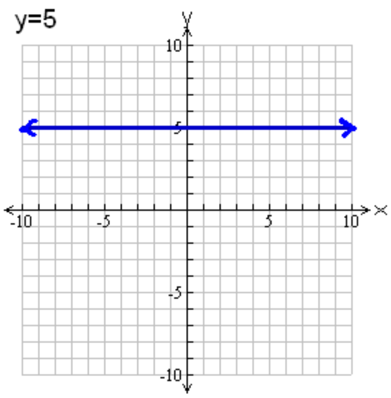


Figure 1

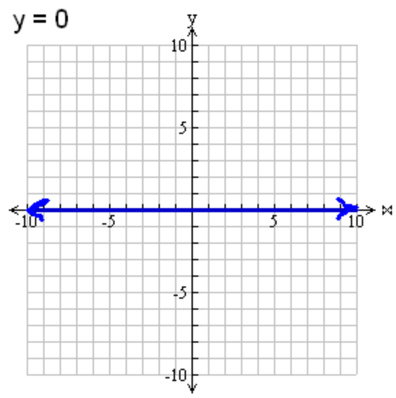


Figure 2

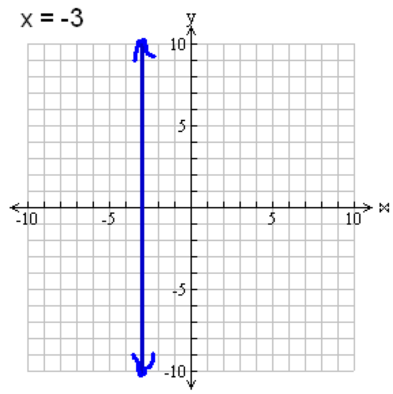


Figure 3

Form 2: Vertical Line $x = c\,(\text{c} = \text{real number})$

The graph of $x = c$ is a vertical line that passes through the point $(c,\ 0)$. The equation $x = -3$ is simply graphed by moving on the x-axis to the point $(-3,\ 0)$ and then drawing a vertical line (see figure 3). Likewise, the equation $x = 0$ is graphed by going to the point $(0,\ 0)$ and drawing a vertical line – in this case, the graph is the y-axis.

Form 3: Slope-Intercept $y = mx + b$ (m = slope, b = y-intercept)

The equation $y = 2x - 3$ is in Slope-Intercept form. To graph this line, first plot the *y*-intercept $(0,\ -3)$. Then, recalling that the slope is the ratio of rise/run (2/1), find a second point on the line by going up 2 and then right 1 [to point $(1,\ -1)$]. Connect the points to graph the line (see figure 4).

http://www.brightstorm.com/math/algebra/linear-equations-and-their-graphs/how-to-graph-a-line-using-y-equals-mx-plus-b
http://www.khanacademy.org/video/graphs-using-slope-intercept-form?playlist=ck12.org%20Algebra%201%20Examples

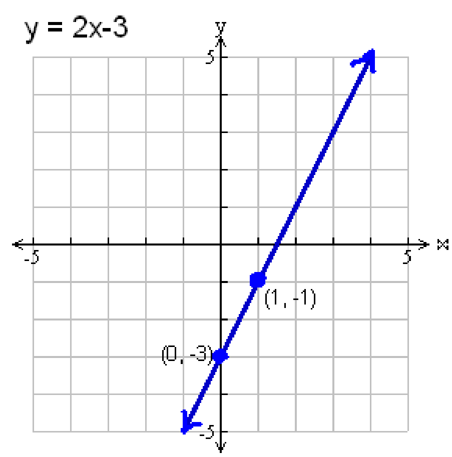


Figure 4

Form 4: Point-Slope $y - y_1 = m(x - x_1)$ [(x_1, y_1) = point, m = slope]

The equation $y - 2 = -2(x - 4)$ is in Point-Slope form. To graph this line, first plot the point $(x_1, y_1) = (4, 2)$. Then, using the slope (-2/1), find a second point on the line by going down 2 and then right 1 [to point $(5, 0)$]. Connect the points to graph the line (see figure 5).

Now, consider the equation $y - 2 = -2(x + 4)$, also in Point-Slope form. In this case, the point $(x_1, y_1) = (-4, 2)$ because the equation is really $y - 2 = -2(x - (-4))$. Use the slope (-2/1) to find a second point on the line by going down 2 and right 1 [to point $(-3, 0)$]. Connect the points to graph the line (see figure 6).

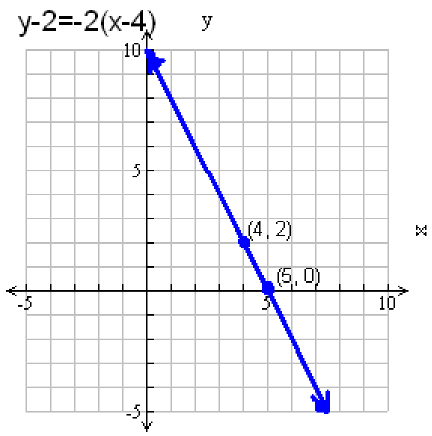


Figure 5

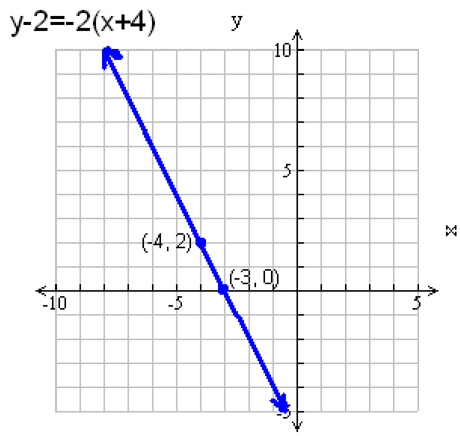


Figure 6

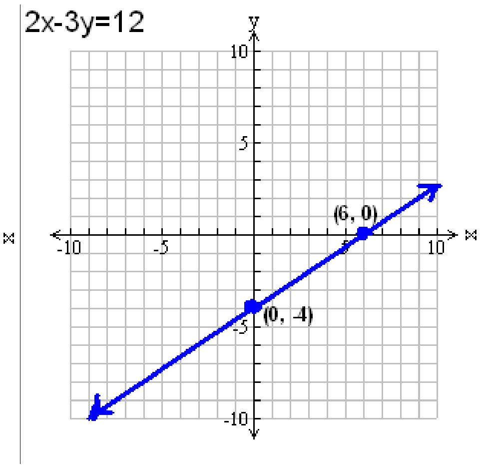


Figure 7

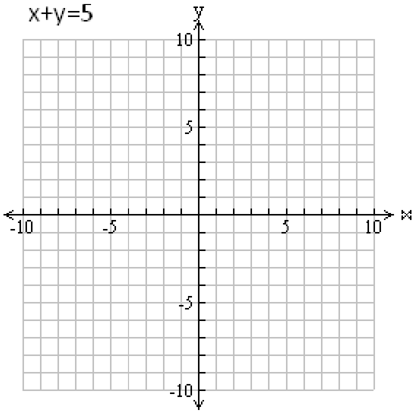
<u>Form 5: Standard Form</u> $Ax + By = C$

The equation $2x - 3y = 12$ is in Standard form. To graph this equation you may be tempted to transform this equation into its equivalent Slope-Intercept form $y = mx + b$. Don't do it! Just find two points, the x- and y-intercepts, and connect with a line. To find the x-intercept, replace y in the equation with 0 and solve for x. This gives you $2x = 12$ or $x = 6$. So, the x-intercept is $(6, 0)$. To find the y-intercept, replace x in the equation with 0 and solve for y. This gives $-3y = 12$ or $y = -4$. So, the y-intercept is $(0, -4)$. Plot and connect these intercepts to obtain the graph (see figure 7).

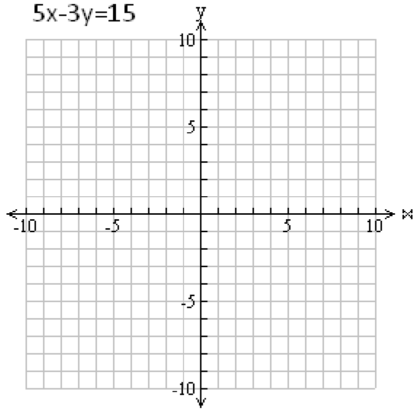
http://brightstorm.com/math/algebra/linear-equations-and-their-graphs/standard-form-of-linear-equations (see Problem 3)

<u>Exercises</u>: Graph each line in the space provided.

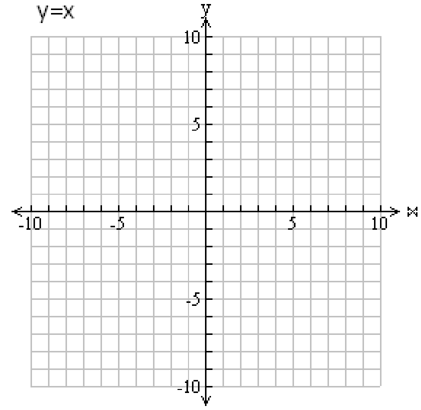
x+y=5



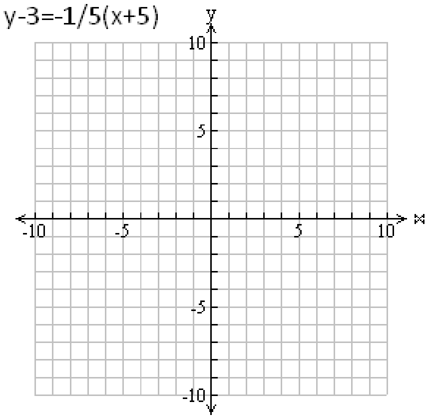
5x-3y=15



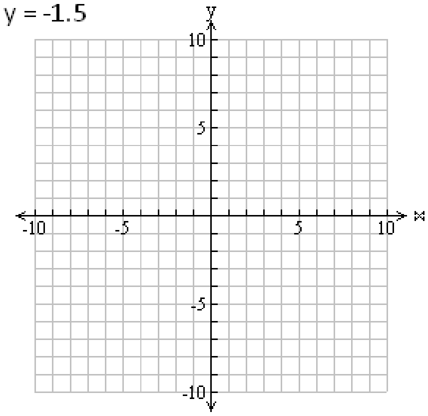
y=x



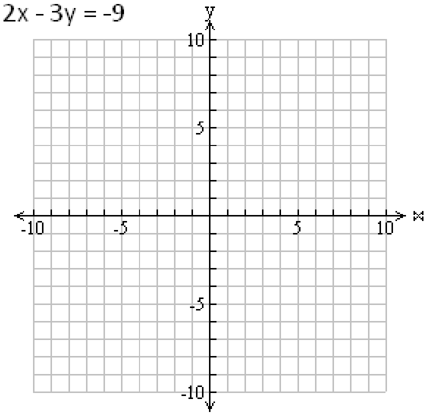
y-3=-1/5(x+5)



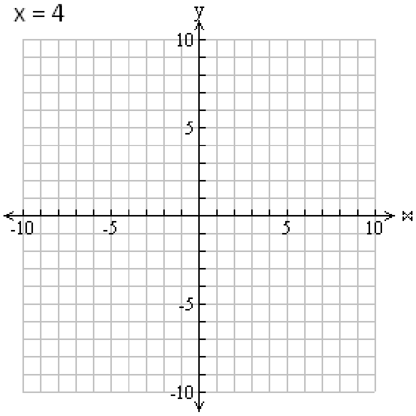
y = -1.5



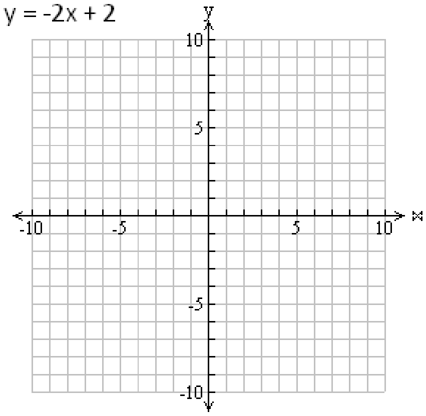
2x - 3y = -9



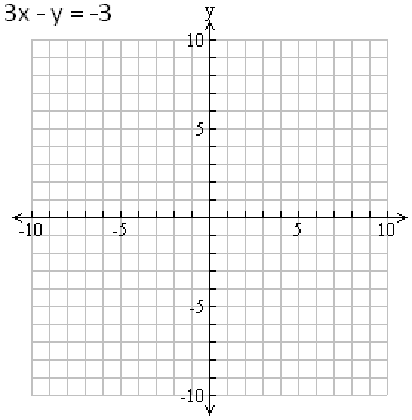
x = 4



y = -2x + 2



3x - y = -3



B. Finding Equations of a Line.

http://brightstorm.com/math/algebra/linear-equations-and-their-graphs/writing-equations-in-slope-intercept-form
http://www.khanacademy.org/video/linear-equations-in-slope-intercept-form?playlist=ck12.org%20Algebra%201%20Examples
http://www.khanacademy.org/video/linear-equations-in-point-slope-form?playlist=ck12.org%20Algebra%201%20Examples

Given a slope and a point, find an equation of a line:

Ex7) Find an equation of the line with a slope of $\frac{2}{3}$ and containing the point $(6,-3)$.

Method I: Since the slope and a point are provided, use the Point-Slope form: $y-y_1=m(x-x_1)$

Step 1: Substitute for the given slope and point: $y-(-3)=\frac{2}{3}(x-6)$

Step 2: Simplify: $y+3=\frac{2}{3}(x-6)$

Method II: Use the Slope-Intercept form $y=mx+b$

Step 1: Substitute for the given slope: $y=\left(\frac{2}{3}\right)x+b$

Step 2: Find b (y-intercept) by substituting in the point:

$$-3=\left(\frac{2}{3}\right)(6)+b$$
$$-3=4+b$$
$$-7=b$$

Step 3: Substitute for b into the equation from Step 1 and simplify: $y=\frac{2}{3}x+(-7)=\frac{2}{3}x-7$

Given two points, find an equation of a line:

Ex8) Find an equation of a line containing the points $(-3,2)$ and $(7,4)$

Step 1: Find the slope: $m=\frac{\Delta y}{\Delta x}=\frac{y_2-y_1}{x_2-x_1}=\frac{4-2}{7-(-3)}=\frac{2}{10}=\frac{1}{5}$

Step 2: Use either **Method I** or **Method II** from Example 7 above. You may use either of the two given points.

Method I (using 1st point).

$$y-(2)=\frac{1}{5}(x-(-3))$$
$$y-2=\frac{1}{5}(x+3)$$

Method II (using 1st point).

$$y=\left(\frac{1}{5}\right)x+b$$
$$2=\frac{1}{5}(-3)+b$$
$$2=-\frac{3}{5}+b$$
$$2+\frac{3}{5}=b$$
$$\frac{10}{5}+\frac{3}{5}=b$$
$$\frac{13}{5}=b \qquad \text{so, } y=\frac{1}{5}x+\frac{13}{5}$$

*NOTE: The above two linear equations may appear different; but, they are equivalent.

For the Method I equation: solve for y -- distribute the 1/5 inside the parenthesis, then add 2 to the right side

$$y-2=\frac{1}{5}(x+3) \Rightarrow y-2=\frac{1}{5}x+\frac{3}{5}$$
$$y=\frac{1}{5}x+\frac{3}{5}+2 \Rightarrow y=\frac{1}{5}x+\frac{3}{5}+\frac{10}{5}$$
$$y=\frac{1}{5}x+\frac{13}{5}$$

Find an equation of the line with the given:

1. Slope of 2 and y-intercept of 1

2. Contains points $(4, -1)$ and $(1, 2)$

3. Slope of -4 and point $(-6, 1)$

4. Slope $\frac{3}{4}$ and a point $(4, 1)$

5. Contains points (5, 6) (5, -3).

4. Graphing Linear Inequalities

Graphing Linear Inequalities in One Variable (in coordinate plane)

http://www.brightstorm.com/math/algebra/solving-and-graphing-inequalities/graphing-2-variable-inequalities
http://www.khanacademy.org/video/graphing-inequalities-2?playlist=Developmental%20Math%202

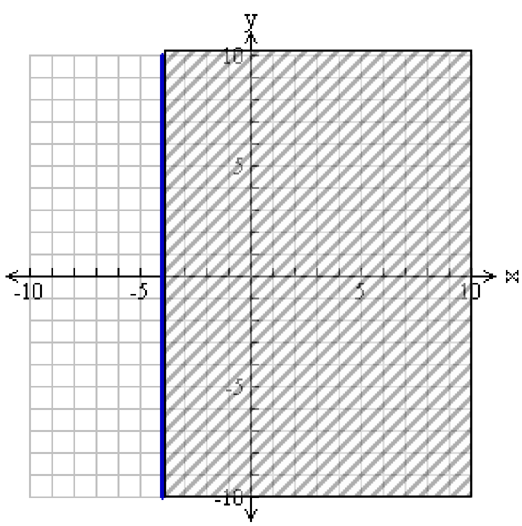
Step 1. Graph the Boundary Line: $x = k$ (k = constant) will be a vertical line; $y = k$ will be a horizontal line.

Step 2. Use "dashed line" if < or >; use "solid line" if ≤ or ≥

Step 3. Shade right (of vertical boundary line) or above (of horizontal boundary line) if > or ≥

Step 4. Shade left (of vertical boundary line) or below (of horizontal boundary line) if < or ≤

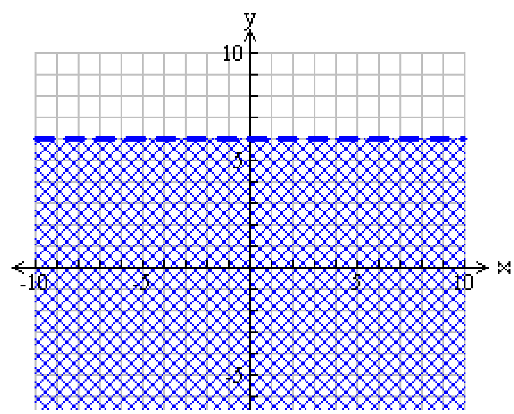
Ex1) Graph $x \geq -4$



Ex2) Graph $3y < 18$

Solve for y: $\frac{3y}{3} < \frac{18}{3}$

$y < 6$



Graphing Linear Inequalities in Two Variables

http://www.brightstorm.com/math/algebra/solving-and-graphing-inequalities/graphing-2-variable-inequalities
http://www.khanacademy.org/video/graphing-inequalities-2?playlist=Developmental%20Math%202

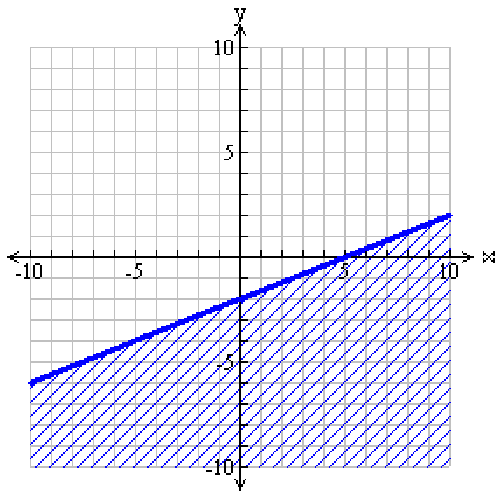
Step 1. Solve the inequality so y is isolated on left side of inequality sign
Step 2. Plot the y intercept and find another point on boundary line using slope
Step 3. Draw boundary line (use "dashed line" if < or >; use "solid line" if ≤ or ≥)
Step 4. Shade above boundary line if > or ≥
Step 5. Shade below boundary line if < or ≤

Ex3) Graph $2x-5y\geq 10$

$$-5y\geq -2x+10$$

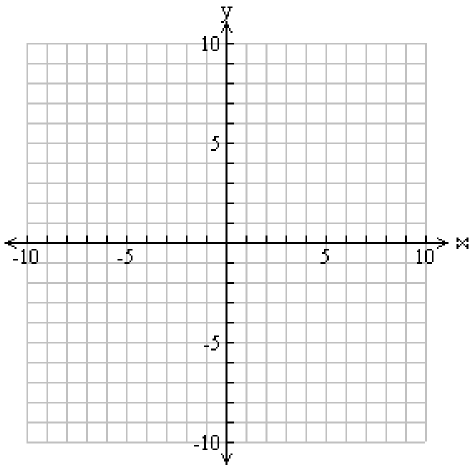
$$y\leq \frac{-2}{-5}x+\frac{10}{-5}$$

$$y\leq \frac{2}{5}x-2$$

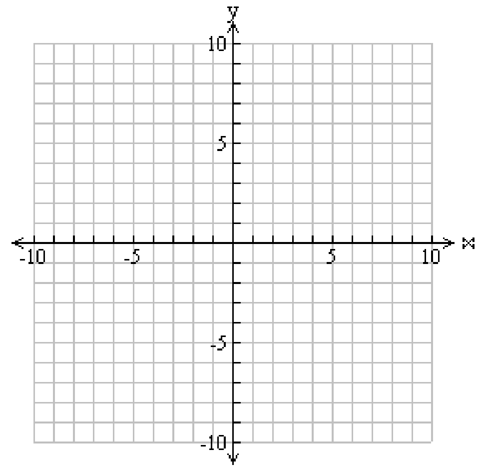


Graph the following inequalities:

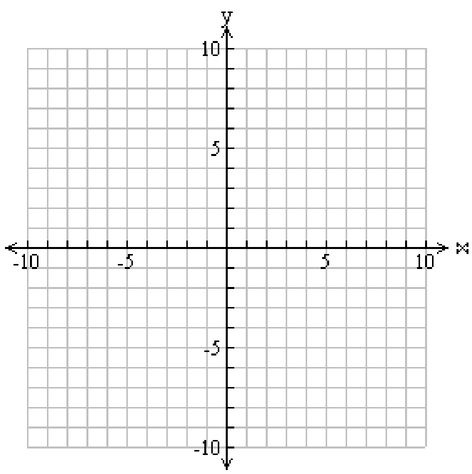
1. $y < 2x - 1$



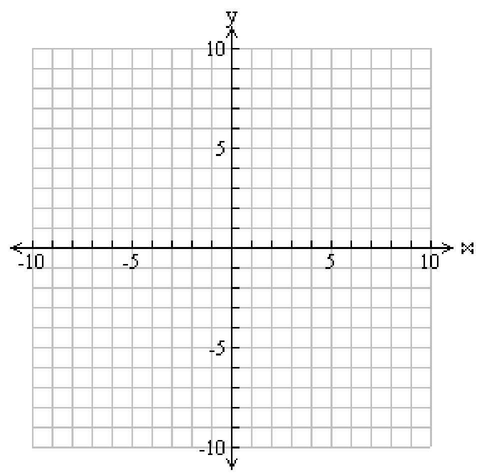
2. $y \geq 2x - 4$



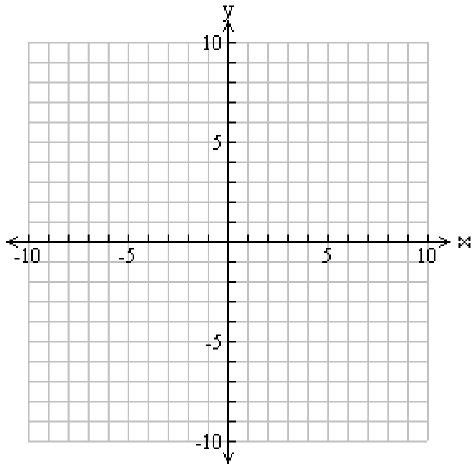
3. $4x + 2y \geq 8$



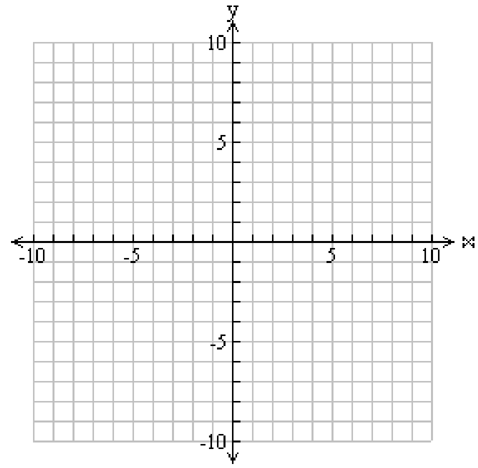
4. $3x - y < 3$



5. $y \geq 2$

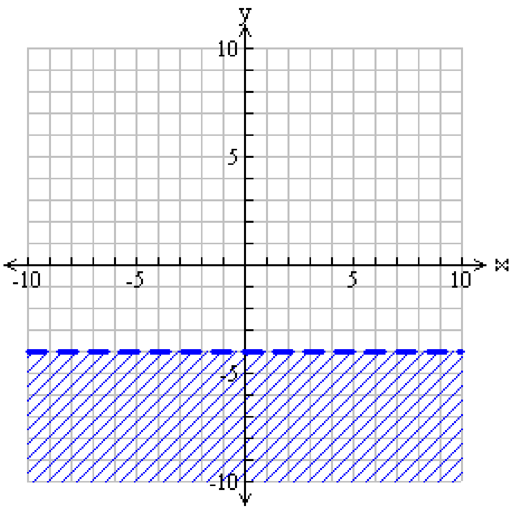


6. $x < 4$

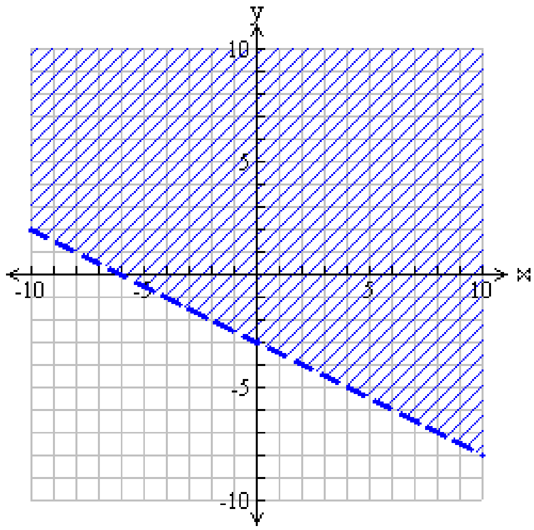


Write the inequality for the graphs:

7.



8.



5. Polynomials – Add/Subtract, Multiply

Suggested videos on polynomials:
http://brightstorm.com/math/algebra/polynomials-2
http://brightstorm.com/math/algebra-2/polynomials/dividing-polynomials-using-long-division
http://www.khanacademy.org/video/addition-and-subtraction-of-polynomials?playlist=ck12.org%20Algebra%201%20Examples
http://www.khanacademy.org/video/multiplication-of-polynomials?playlist=ck12.org%20Algebra%201%20Examples
http://www.khanacademy.org/video/polynomial-division?playlist=ck12.org%20Algebra%201%20Examples

Addition: Add *like* terms

Ex1)

$$\left(4x+6y\right)+\left(2x-3y\right)=$$
$$\left(4x+2x\right)+\left(6y-3y\right)=$$
$$6x+3y$$

Subtraction: Add the *additive inverse*

Ex2)

$$\left(x^3+2x^2-8x\right)-\left(-2x^2+7x-5\right)=$$
$$\left(x^3+2x^2-8x\right)+\left(+2x^2-7x+5\right)=$$
$$\left(x^3\right)+\left(2x^2+2x^2\right)+\left(-8x-7x\right)+\left(5\right)=$$
$$x^3+4x^2-15x+5$$

Multiplication: Use the distributive property.

Ex3)

$$7y(-6y-9)=7y(-6y)+7y(-9)$$
$$=-42y^2-63y$$

Ex4)

$$(x+2)(x-5)=x^2-5x+2x-10$$
$$=x^2-3x-10$$

***Recall FOIL (First-Outside-Inside-Last)**

SIMPLIFY: Circle answers

1. $(y^2+2y-5)+(8y^2-5y+9)$

2. $(7x^3-5x^2-2)-(5x^3-2x^2+4)$

3. $-2xy(6x^2-4xy+5y^2)$

4. $(2x-5)(3x+2)$

5. $(2x+3)(3x+5)$

6. $(3x-5)^2$

6. Factoring Polynomials

Suggested videos on factoring polynomials:
http://brightstorm.com/math/algebra/factoring-2
http://www.khanacademy.org/video/factoring-quadratic-expressions?playlist=ck12.org%20Algebra%201%20Examples

Perfect Square Trinomials $a^2+2ab+b^2=(a+b)^2$ and $a^2-2ab+b^2=(a-b)^2$
http://brightstorm.com/math/algebra/factoring-2/factoring-special-cases-part-i

Ex1) $x^2+4x+4=(x+2)(x+2)=(x+2)^2$

Ex2) $y^2-14y+49=(y-7)(y-7)=(y-7)^2$

Ex3) $4x^2-4xy+y^2=(2x-y)(2x-y)=(2x-y)^2$

Factor the following perfect square trinomials:

1. x^2+6x+9

2. $y^2-16y+64$

3. $x^2+4xy+4y^2$

4. $9x^2-12xy+4y^2$

Trinomials $ax^2+bx+c,\ a=1$ (i.e., Lead Coefficient =1) Examples: $x^2+8x+15$ and x^2-4x+3

http://brightstorm.com/math/algebra/factoring-2/factoring-trinomials-a-equals-1

Example: Factor $x^2+2x-15$

Step 1: Find the square root of the first term. The square root of the first term is *x*.

Step 2: Find all factors of the third term, -15. The factors of the third term are: $\{-3,5\},\{3,-5\},\{-1,15\},\{1,-15\}$

Step 3: Decide which of these factors can be added to find the coefficient of the middle term.

$5+(-3)=2$, so $\{-3,5\}$ are the two factors needed to get the middle term.

Therefore, $(x-3)(x+5)$ are the two binomial factors. Check the answer by FOILing: $(x-3)(x+5)=x^2+2x-15$

Ex4) $x^2+8x+15=(x+5)(x+3)$

Ex5) $x^2-4x+3=(x-3)(x-1)$

Ex6) $y^2+10y-11=(y+11)(y-1)$

Factor the following trinomials:

5. x^2+8x+7

6. $y^2-7y+12$

7. $x^2-7x+10$

8. $y^2+9y-20$

Difference of Two Squares $a^2-b^2=(a+b)(a-b)$ Examples: x^2-16 and $4x^2-25y^2$

http://brightstorm.com/math/algebra/factoring-2/factoring-special-cases-part-i

Example: Factor x^2-y^2

Step 1: Find the square root of each term: $\sqrt{x^2}=x, \sqrt{y^2}=y$

Step 2: The first factor will be the SUM of these two square roots: $(x+y)$

Step 3: The second factor will be the DIFFERENCE of these two square roots: $(x-y)$

Therefore, $x^2-y^2=(x+y)(x-y)$

Ex11) $x^2-16=(x+4)(x-4)$

Ex12) $4x^2-25y^2=(2x-5y)(2x+5y)$

Ex13) $8x^2-32=8(x^2-4)=8(x+2)(x-2)$

Ex14) $4-9x^2=(2-3x)(2+3x)$

Factor the following difference of two squares:

13. x^2-49

14. $81y^2-1$

15. $100-9y^2$

16. $9x^2-64y^2$